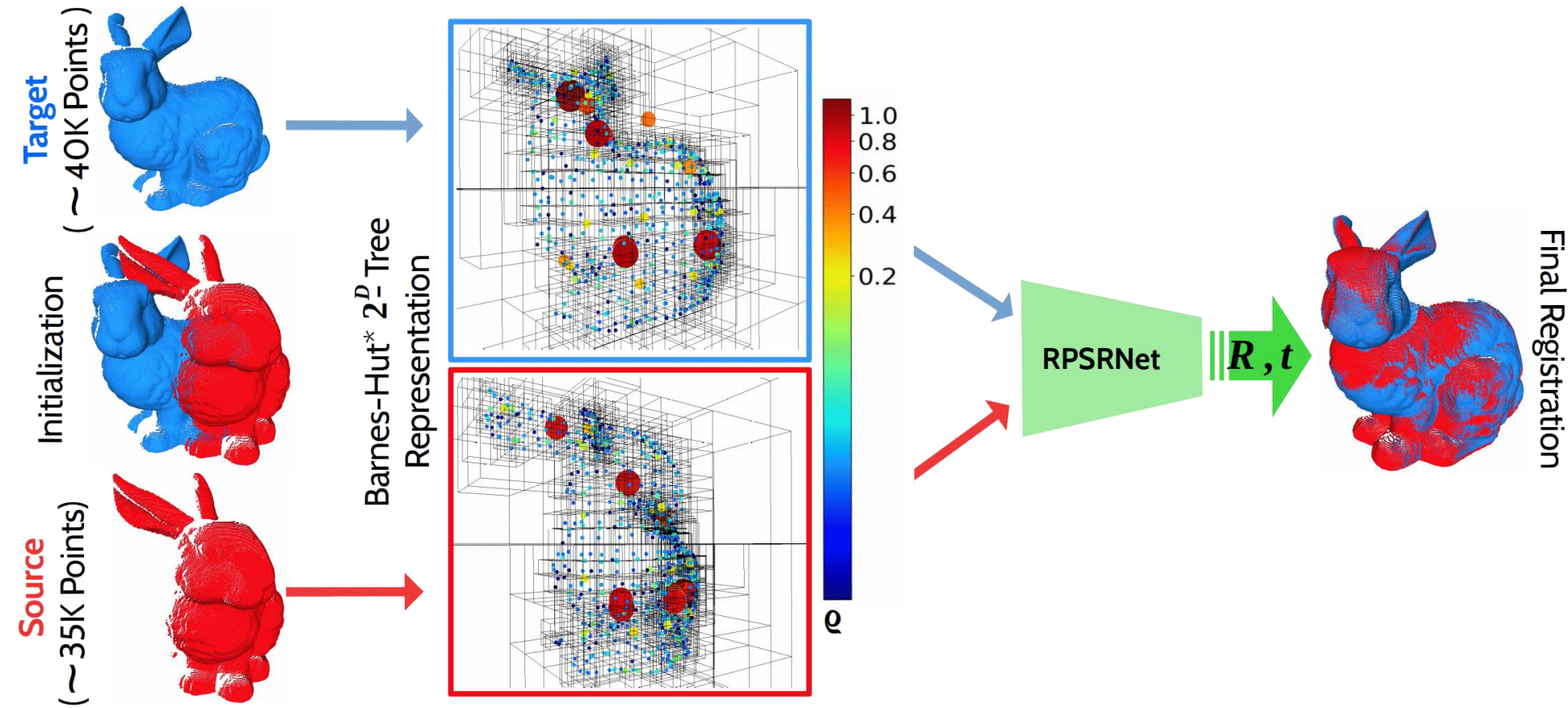


# RPSRNet: End-to-End Trainable Rigid Point Set Registration Network using Barnes-Hut $2^D$ -Tree Representation

Sk Aziz Ali , Kerem Kahraman, Gerd Reis, Didier Stricker  
(TU Kaiserslautern, German Research Center for Artificial Intelligence)



# A supervised deep learning framework (end-to-end trainable network) For Rigid Point Set Registration (RPSR)



# Motivation

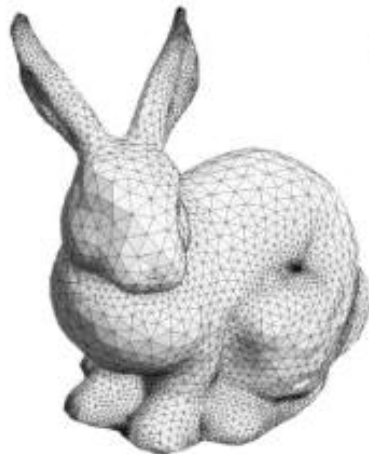
## Input Representations



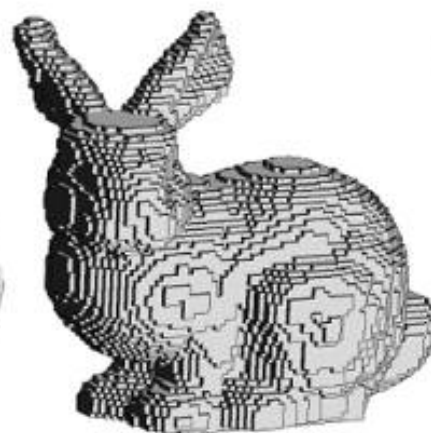
**CAD Model**



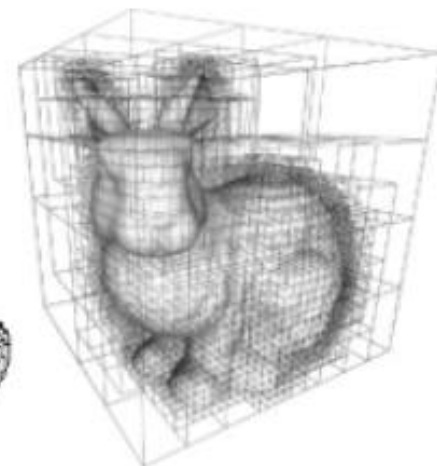
**Dense Point  
Cloud**



**Mesh**



**Voxels**



**Octree**

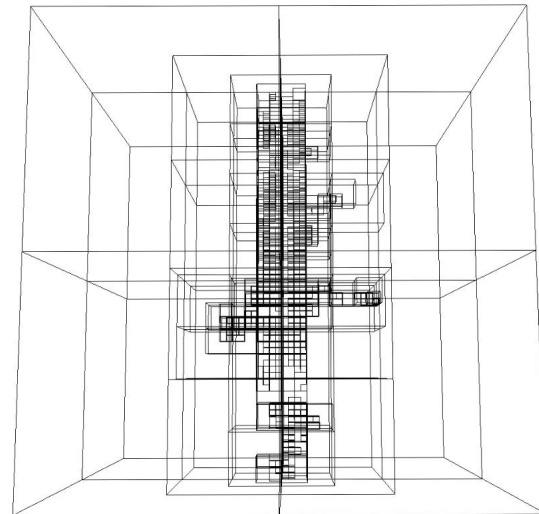
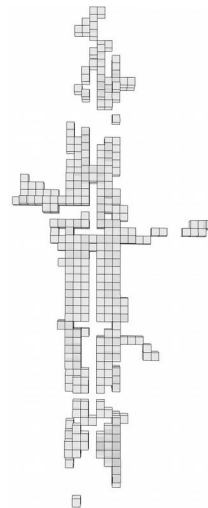
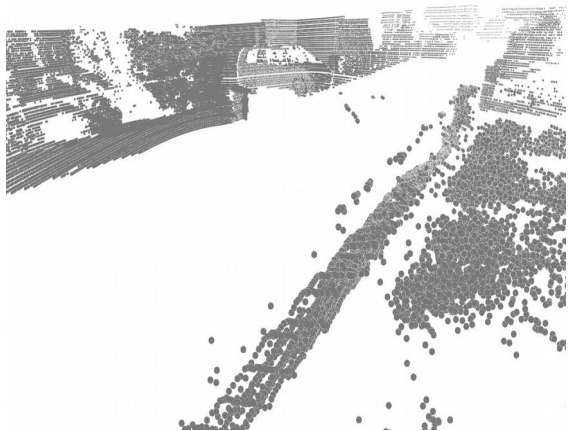
Deep-Learning-Based 3D Point Cloud Processing Tasks

**Classification, Segmentation, Matching/registration, Shape Completion, MODT**

Data Dimension, Memory Efficiency, Shape Details, Computational Efficiency !

# Input Representations

X



CAD Model

Dense Point  
Cloud

BEV  
(Birds Eye View)

Voxels

Octree

Deep-Learning-Based 3D Point Cloud Processing Tasks  
Classification, Segmentation, Matching/registration, Shape Completion, MODT

Data Dimension, Memory Efficiency, Shape Details, Computational Efficiency !

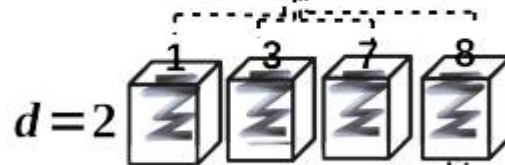
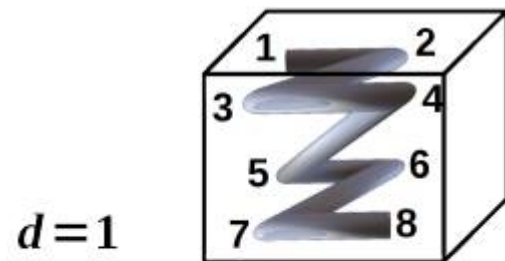
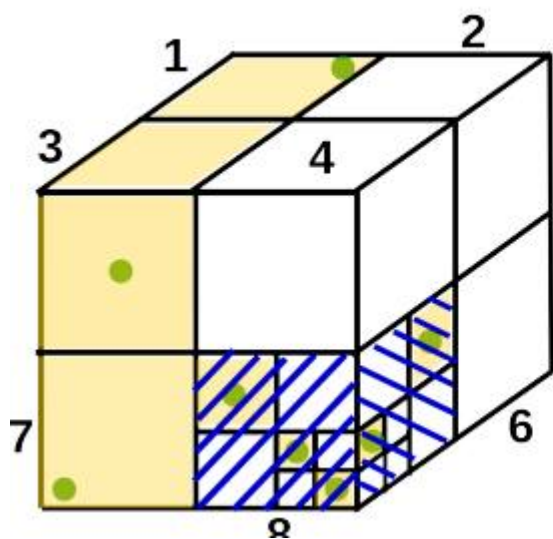
Memory Efficiency	Robust to Noise	Indoor+ Outdoor	Real-time inference	Quick- Training Time	Accept Original Point Size	Partial Data Registration
<sup>1</sup> DCP	<sup>1</sup> DCP		<sup>1</sup> DCP	<sup>1</sup> DCP		<sup>1</sup> DCP
		<sup>2</sup> PointNetLK <sup>3</sup> DGR			<sup>3</sup> DGR	<sup>3</sup> DGR
	<sup>4</sup> RPMNet					<sup>4</sup> RPMNet
<sup>5</sup> PPFFoldNet+ RelativeNet			<sup>5</sup> PPFFoldNet+ RelativeNet		<sup>5</sup> PPFFoldNet+ RelativeNet	<sup>5</sup> PPFFoldNet+ RelativeNet
<sup>6</sup> DeepGMR	<sup>6</sup> DeepGMR		<sup>6</sup> DeepGMR	<sup>6</sup> DeepGMR		
<sup>7</sup> ICP		<sup>7</sup> ICP	<sup>7</sup> ICP	<sup>7</sup> ICP		
<sup>8</sup> CPD	<sup>8</sup> CPD					
<sup>9</sup> FilterReg						<sup>9</sup> FilterReg
	<sup>10</sup> GA					
<sup>11</sup> FGR				<sup>11</sup> FGR	<sup>11</sup> FGR	<sup>11</sup> FGR
RPSRNet	RPSRNet	RPSRNet	RPSRNet	RPSRNet	RPSRNet	RPSRNet

- [1] Y. Wang et. al, **ICCV'19**; [2] Y. Aoki et. al, **CVPR'19**; [3] C. Choy et. al, **CVPR'20**;  
[4] Z. J. Yew et. al, **CVPR'20**; [5] H. Deng et. al, **CVPR'19**; [6] W. Yuan et. al, **ECCV'20**;  
[7] P. J. Besl, **TPAMI'92**; [8] A. Myronenko, **TPAMI'10**; [9] W. Gao et. al, **CVPR'19**;  
[10] V. Golyanik et. al, **CVPR'16**; [11] Q. Y. Zhou et. al, **ECCV'16**

# Input Representation & RPSRNet Framework



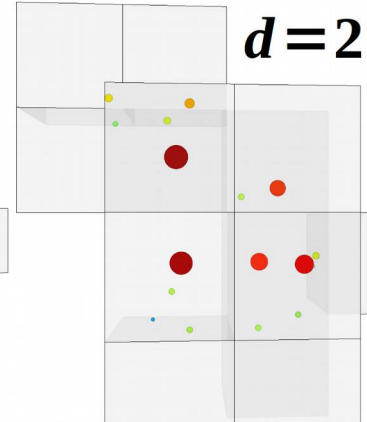
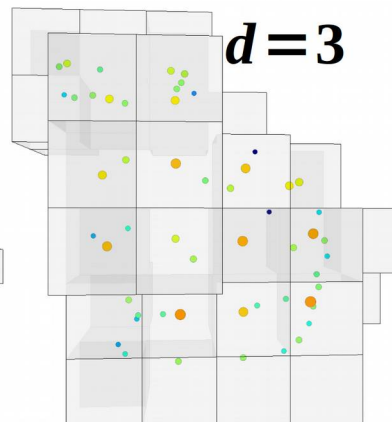
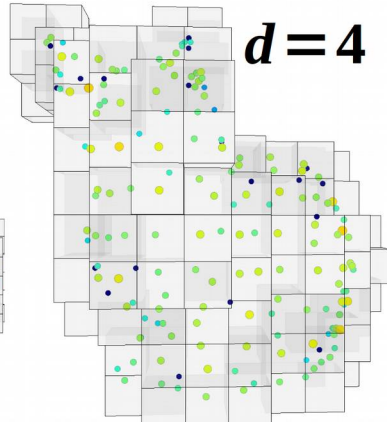
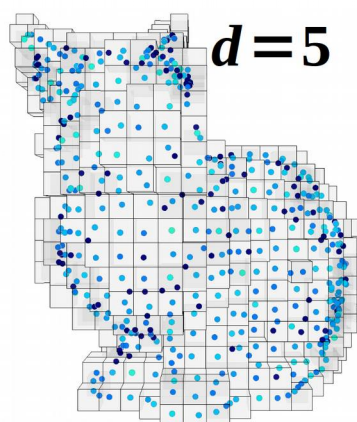
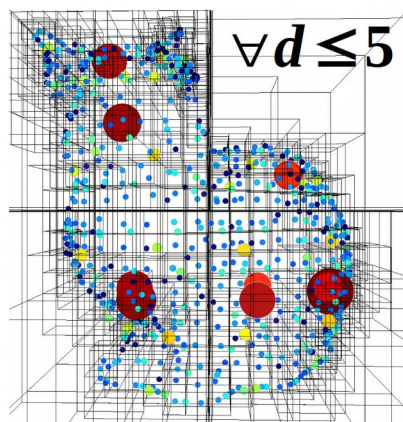
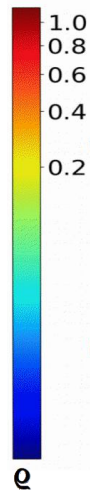
# BH-Tree Node Partitioning and Tree Traversal Indexing



'Leaf' Node
  'Internal' Node
  'Null' Node

'Morton's Z' curve to index  $(1, 2, \dots, 2^3)$  non-empty nodes for Parent-Child relationship





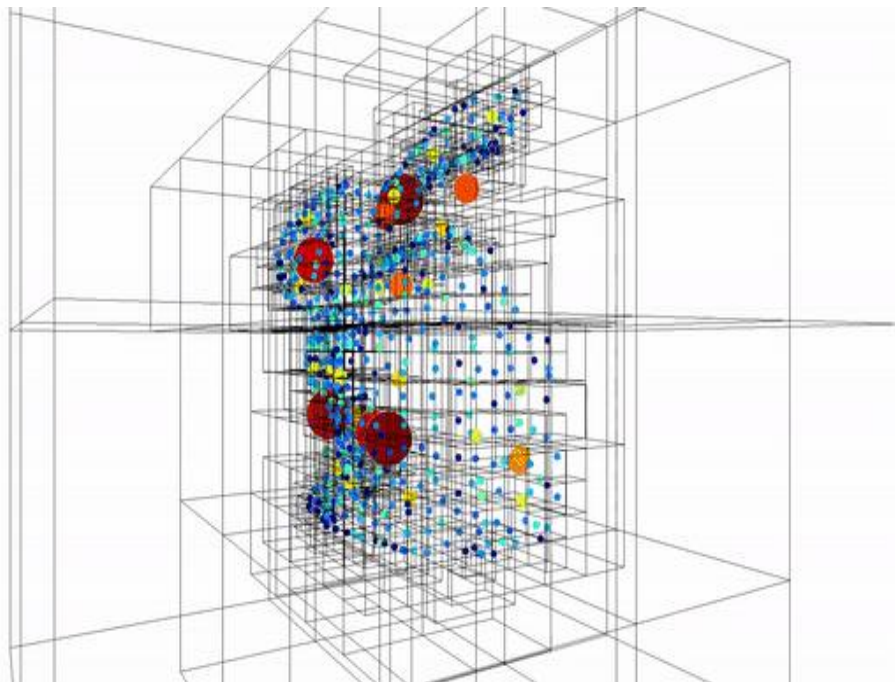
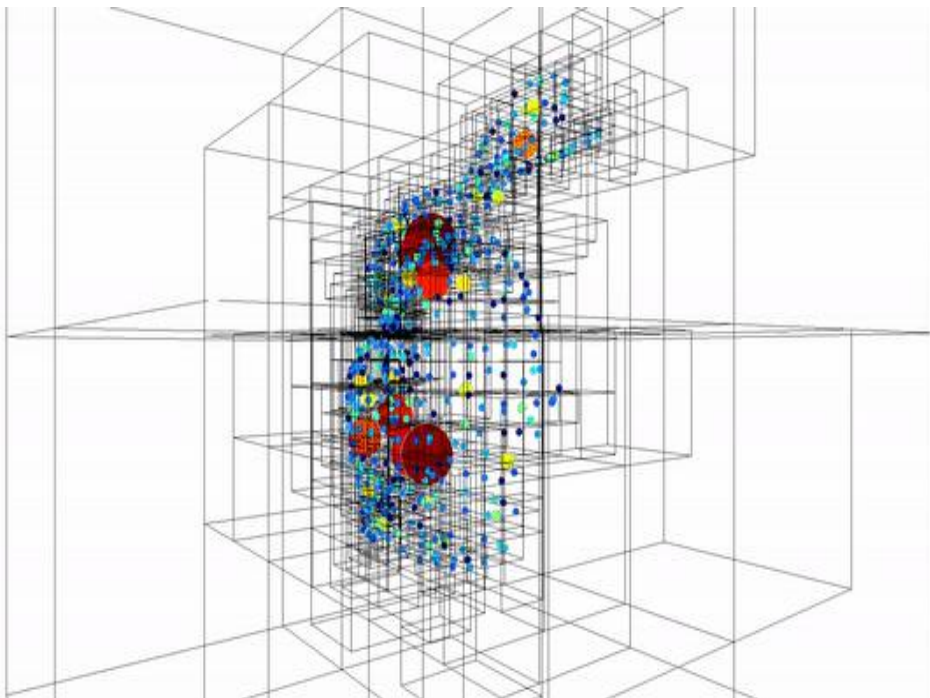
BH-Tree Nodes at every depth

The RPSR problem is often formulated as optimization of cost function  $\mathbf{U}(\mathbf{R}, \mathbf{t})$  in the form of globally **multiply-linked** correspondence distance errors between  $\mathbf{X}$  and  $\mathbf{Y}$  :

$$\mathbf{U}(\mathbf{R}, \mathbf{t}, \mathbf{X}, \mathbf{Y}) = \sum_{i,j} \omega_{ij} \|(\mathbf{R}\mathbf{y}_i + \mathbf{t}) - \mathbf{x}_j\|_2^2,$$

$$\begin{aligned} \mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\} \in \mathbb{R}^{N \times 3} &\rightarrow \tau^{\mathbf{X}}, & \mathbf{M}_d^{\mathbf{Y}} = \{\mu_{d,l}^y\}, \rho_d^{\mathbf{X}^-} = \{\varrho_{d,l}^{x^-}\}, & \mathbf{N}_d^{\mathbf{X}} = \{\mathbf{n}_{d,l}^x\} \\ \mathbf{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_M\} \in \mathbb{R}^{M \times 3} &\rightarrow \tau^{\mathbf{Y}}, & \underbrace{\mathbf{M}_d^{\mathbf{X}} = \{\mu_{d,l}^x\}}_{\text{CoMs}}, \underbrace{\rho_d^{\mathbf{Y}^-} = \{\varrho_{d,l}^{y^-}\}}_{\text{IDs}}, & \underbrace{\mathbf{N}_d^{\mathbf{Y}} = \{\mathbf{n}_{d,l}^y\}}_{\text{Node Idx.}} \end{aligned}$$

**multiply-linked** correspondence distance errors are now applicable on the CoMs of the non-empty nodes at every depth (  $\sum_{i,j} \rightarrow \sum_d \sum_{l,\hat{l}}$  )



multi-scale sum of mean-squared distance errors between the CoMs

$$\mathbf{U}(\mathbf{R}, \mathbf{t}, \tau^{\mathbf{X}}, \tau^{\mathbf{Y}}) = \sum_d \sum_{l, \hat{l}} \varrho_{d,l}^{y-} \varrho_{d,\hat{l}}^{x-} \left\| \left( \mathbf{R} \mu_{d,l}^y + \mathbf{t} \right) - \mu_{d,\hat{l}}^x \right\|_2^2$$

**BH-trees**

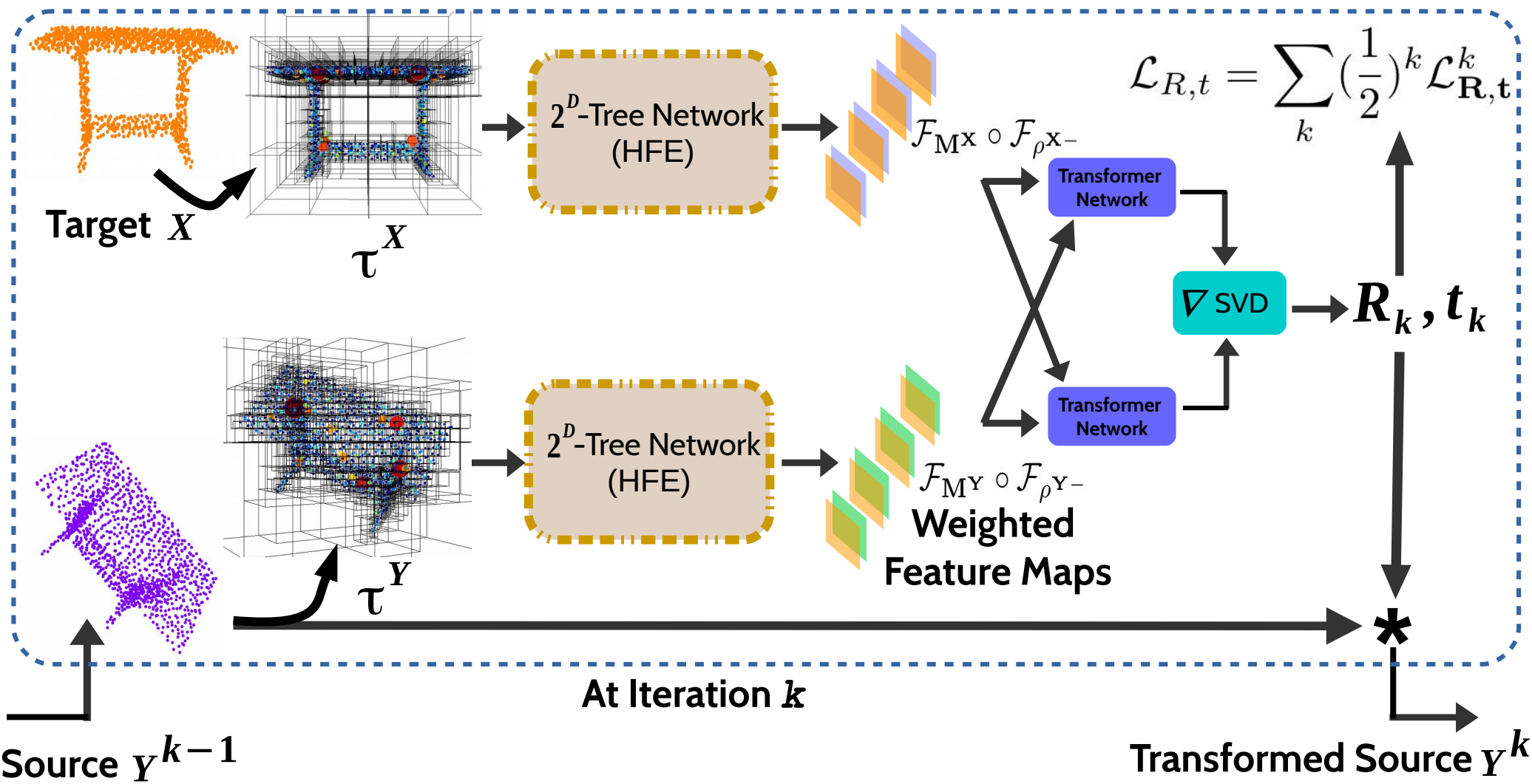
**Depth**

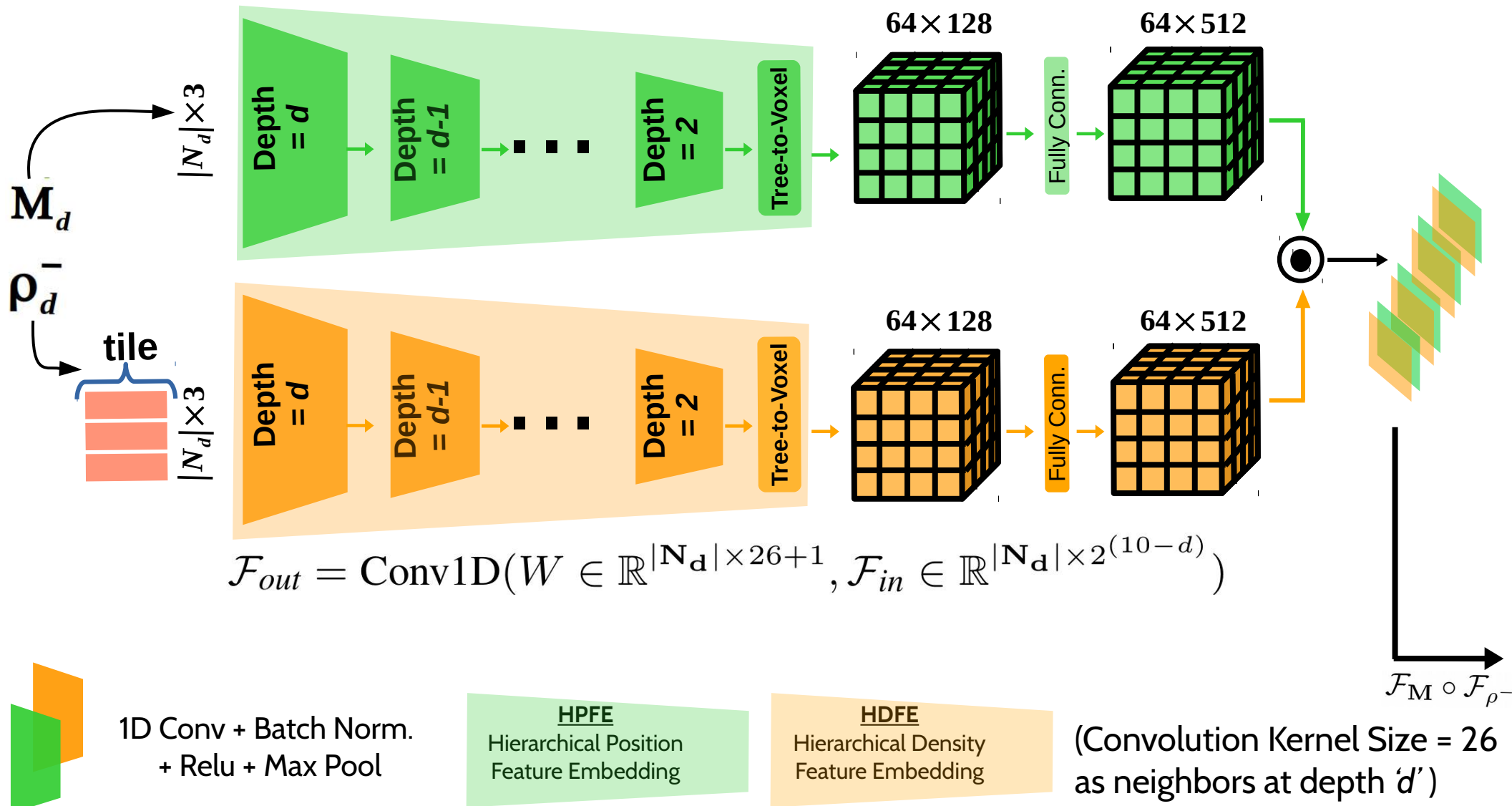
**Node Labels**

**Inverse  
Node Densities**

**Center of Masses  
of Nodes**

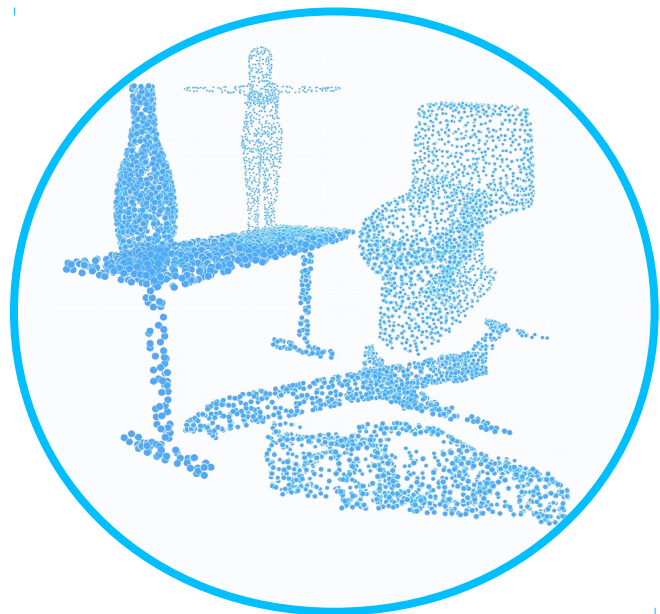
# Architecture of RPSRNet





# Experiments & Results

## ModelNet40 Dataset



**M1-seen**

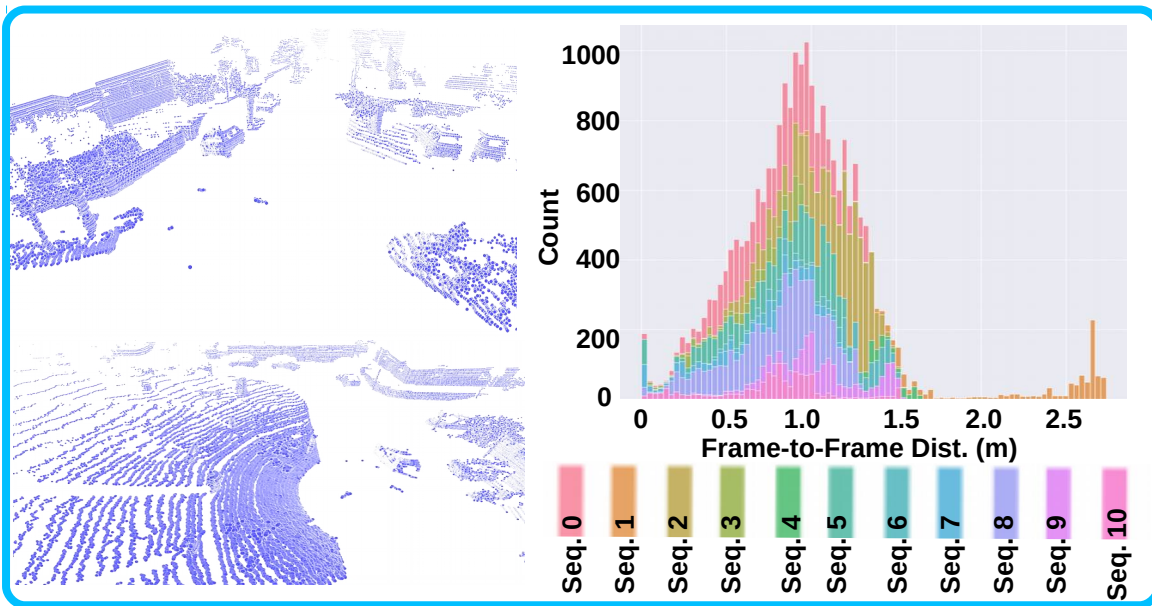
**M2-unseen**

(Data Disturbances: Clean, Jitters, Copping, Gaussian Noise, Uniformly Dist. Noise)

\*Each with 5 increasing levels

$$\varphi = \cos^{-1} (0.5(\text{tr}(\mathbf{R}_{gt}^T \mathbf{R}) - 1)), \quad \Delta \mathbf{t} = \|\mathbf{t}_{gt} - \mathbf{t}\|.$$

## KITTI LiDAR Odometry Dataset



**K1-w/o ground**

**K2-w ground**

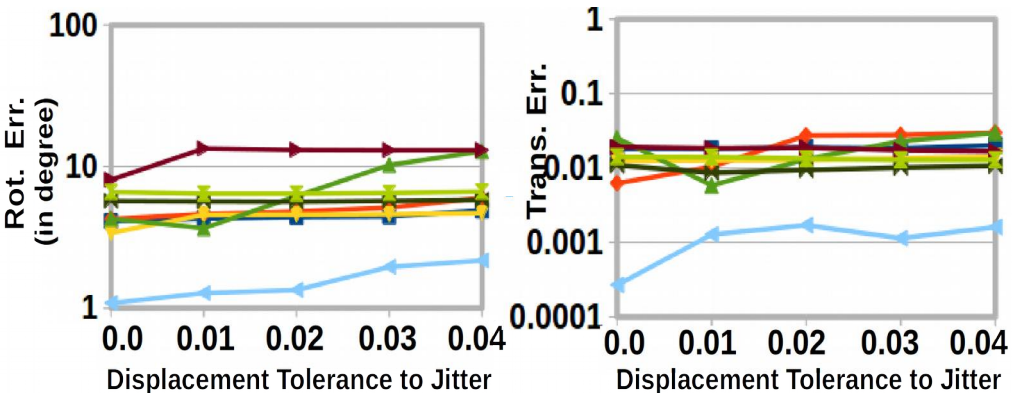
(Train: 70%; Test: 20%; Validation: 10%;)

Of random samples from every Seq.

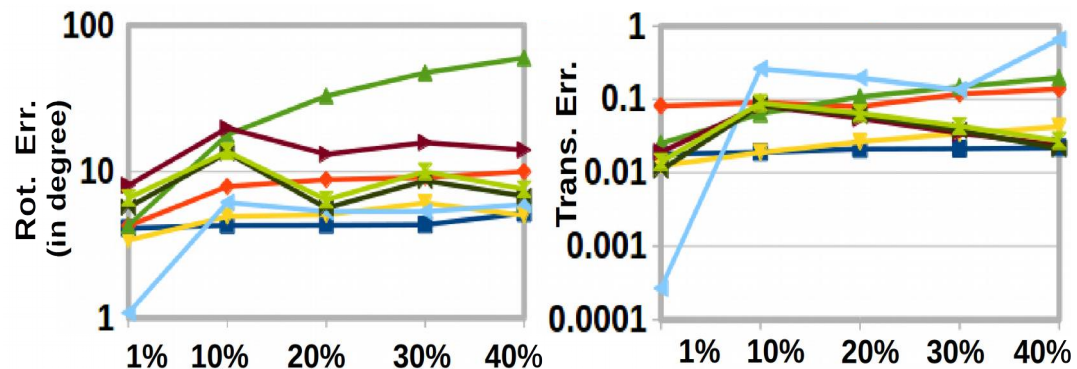


# (M1-seen)

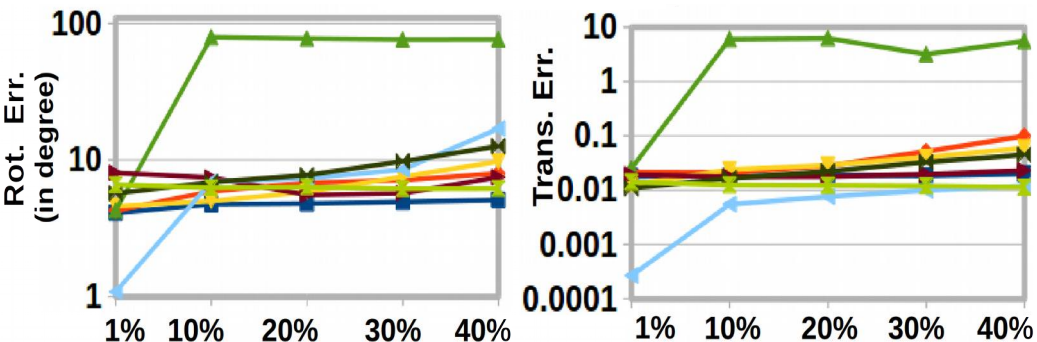
## Perturbation



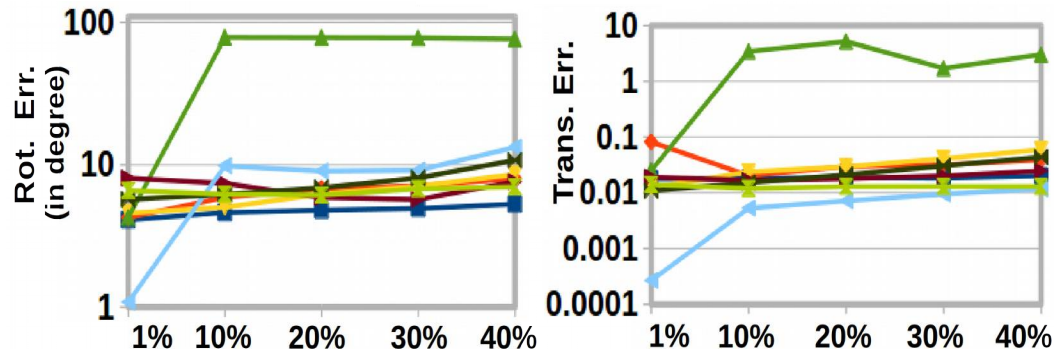
## Cropping



## Gaussian Noise



## Uniform Noise



■ RPSRNet(Ours)

■ DCP-V2 (svd)

▶ GA\* (in GPU)

▶ ICP

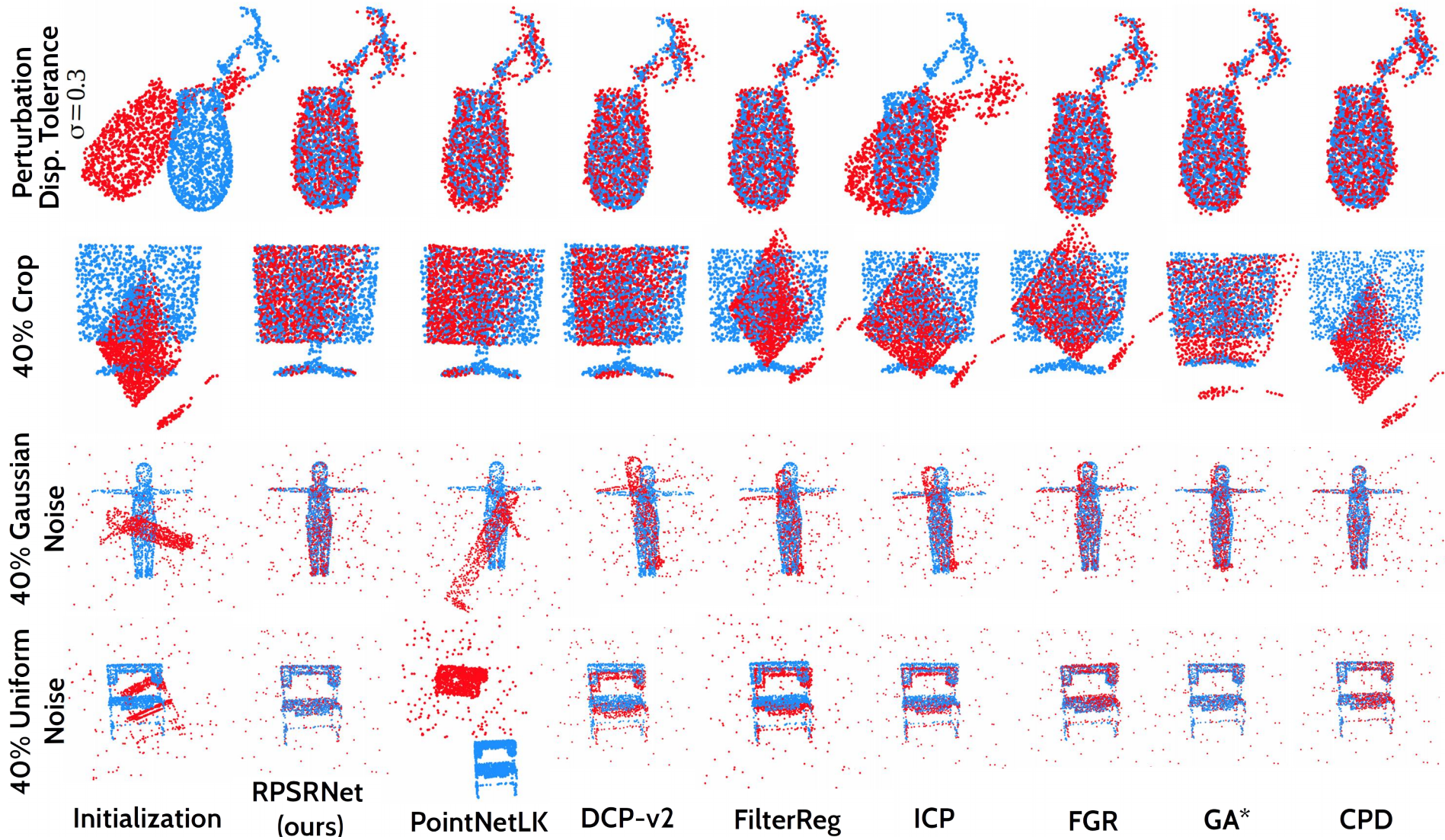
◆ CPD

▲ PointNetLK

▶ FGR

■ FilterReg

# (M1-seen)



(M1-seen)

PointNetLK

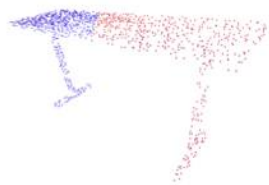
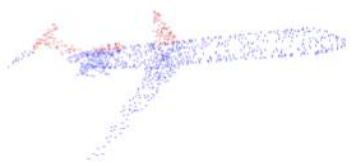
DCP

RPMNet

RPSRNet  
(Ours)

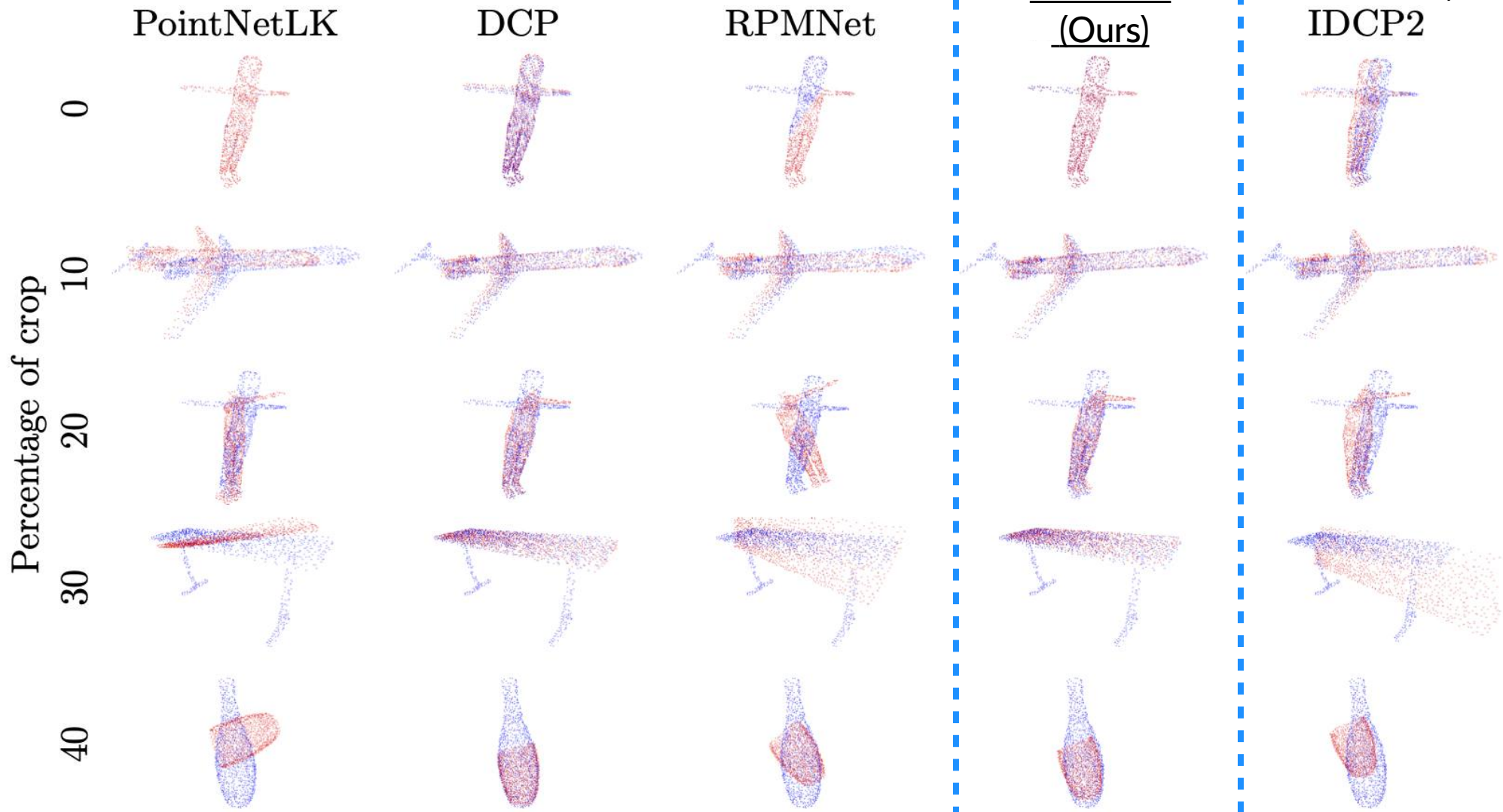
(DCP with Two  
Internal Iter.)  
IDCP2

Clean Data

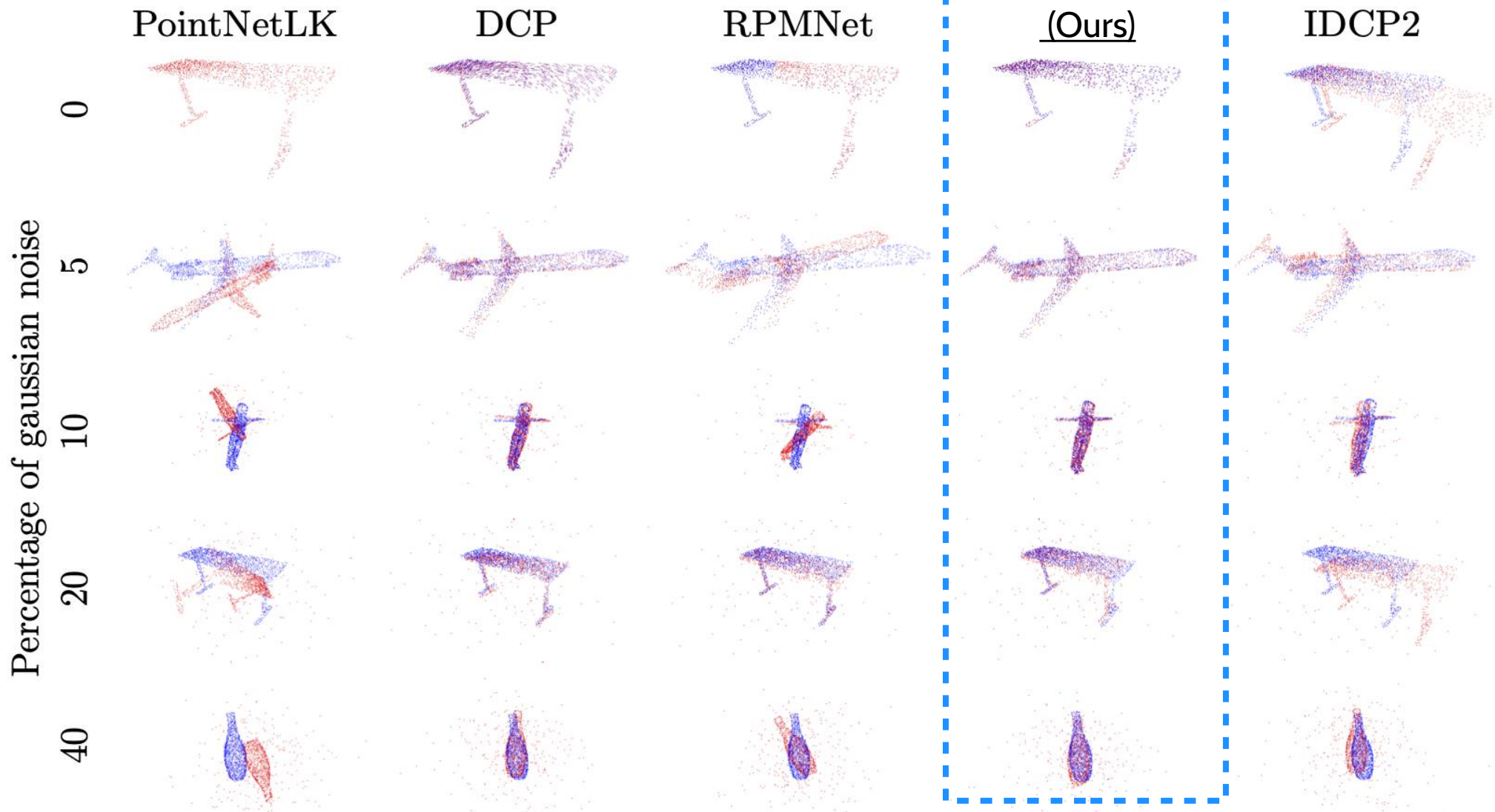


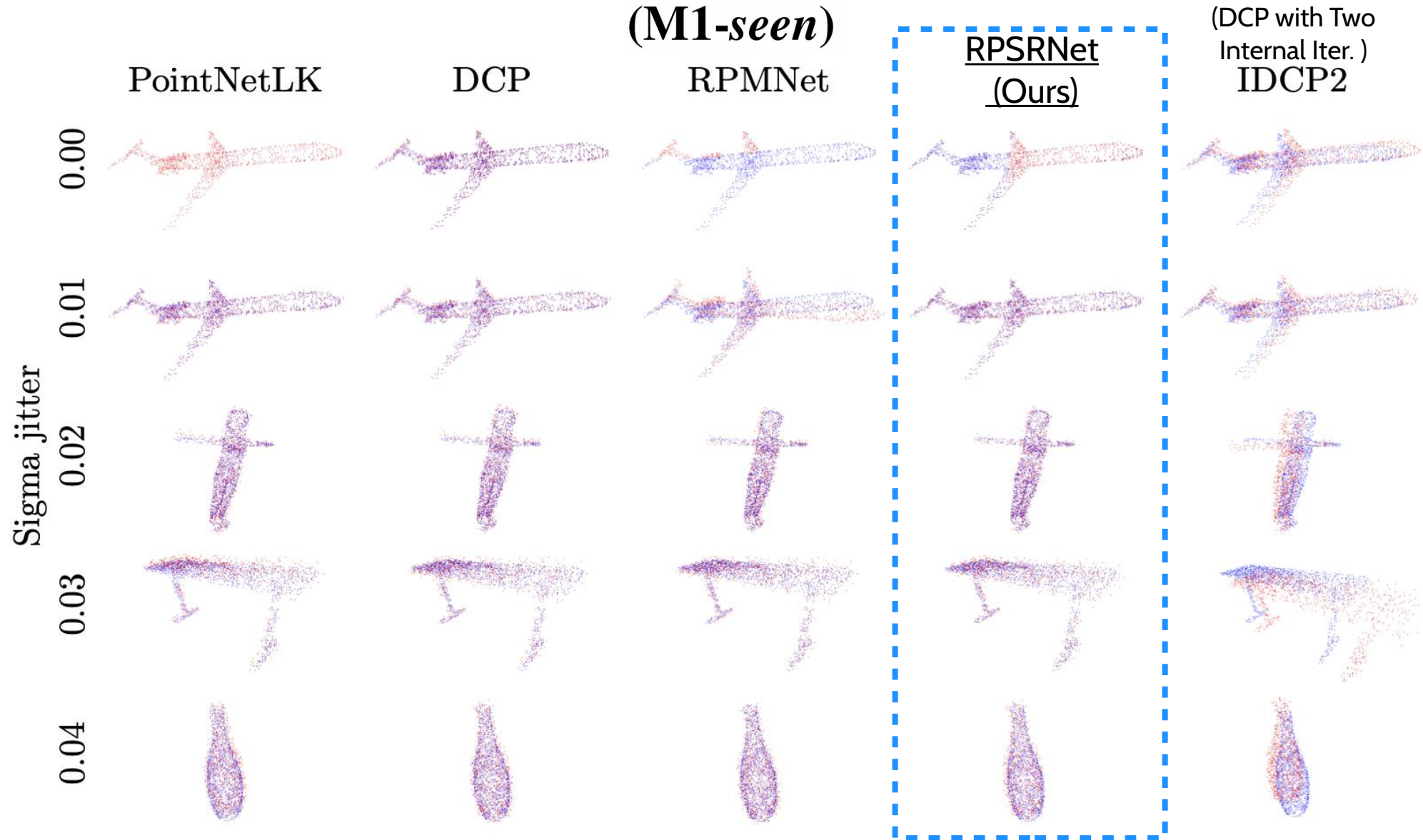


(M1-*seen*)

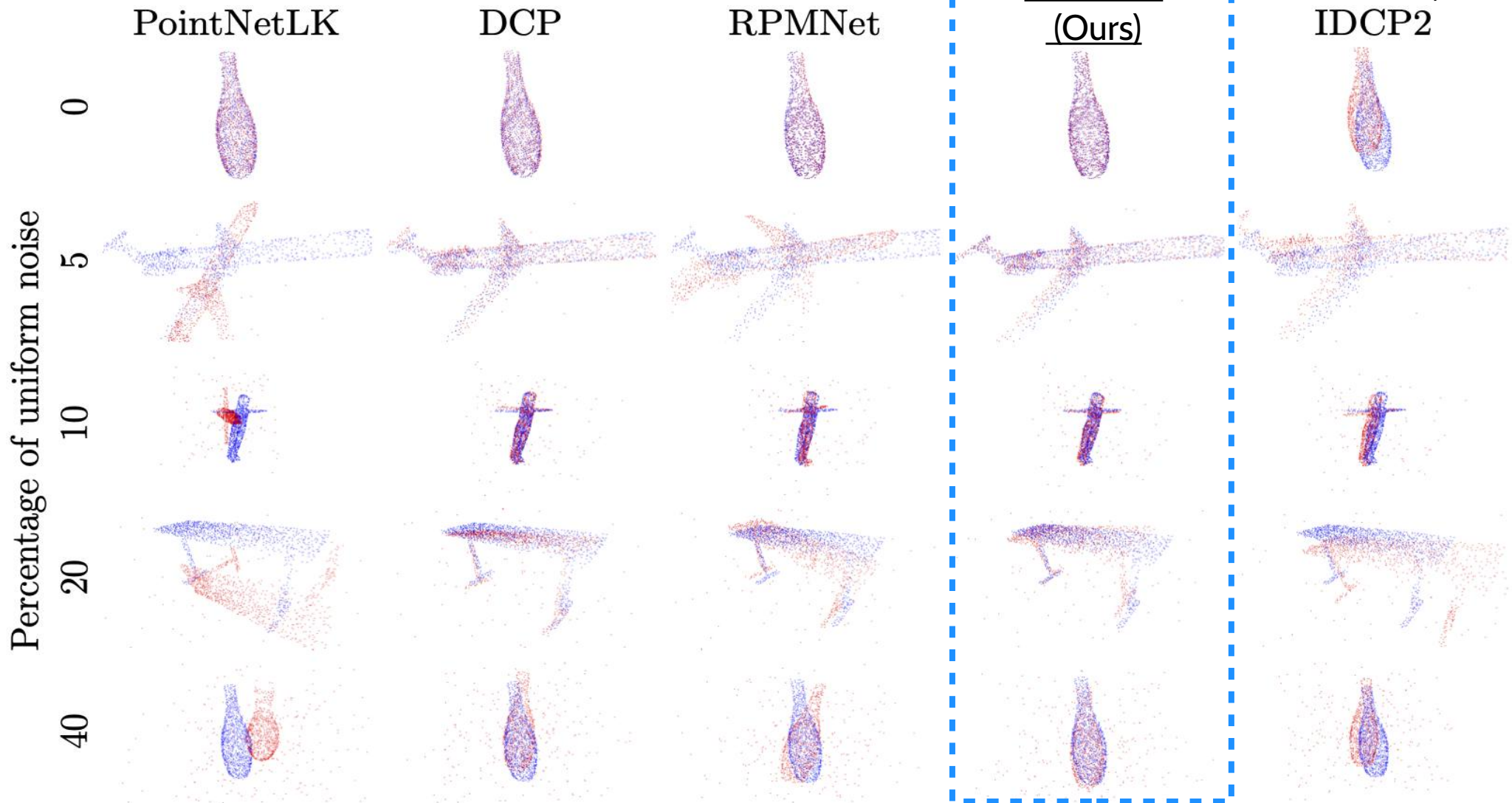


(M1-seen)





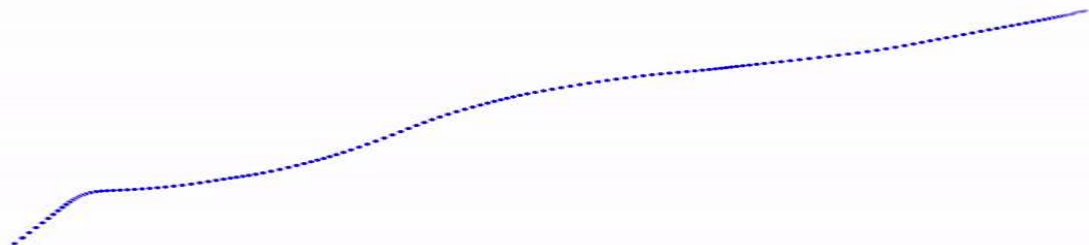
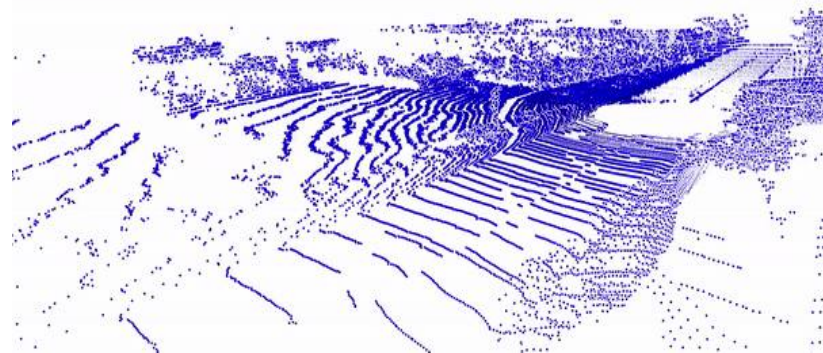
(M1-seen)





# KITTI LiDAR Odometry (Without Pose Graph Optimization and Loop Closure)

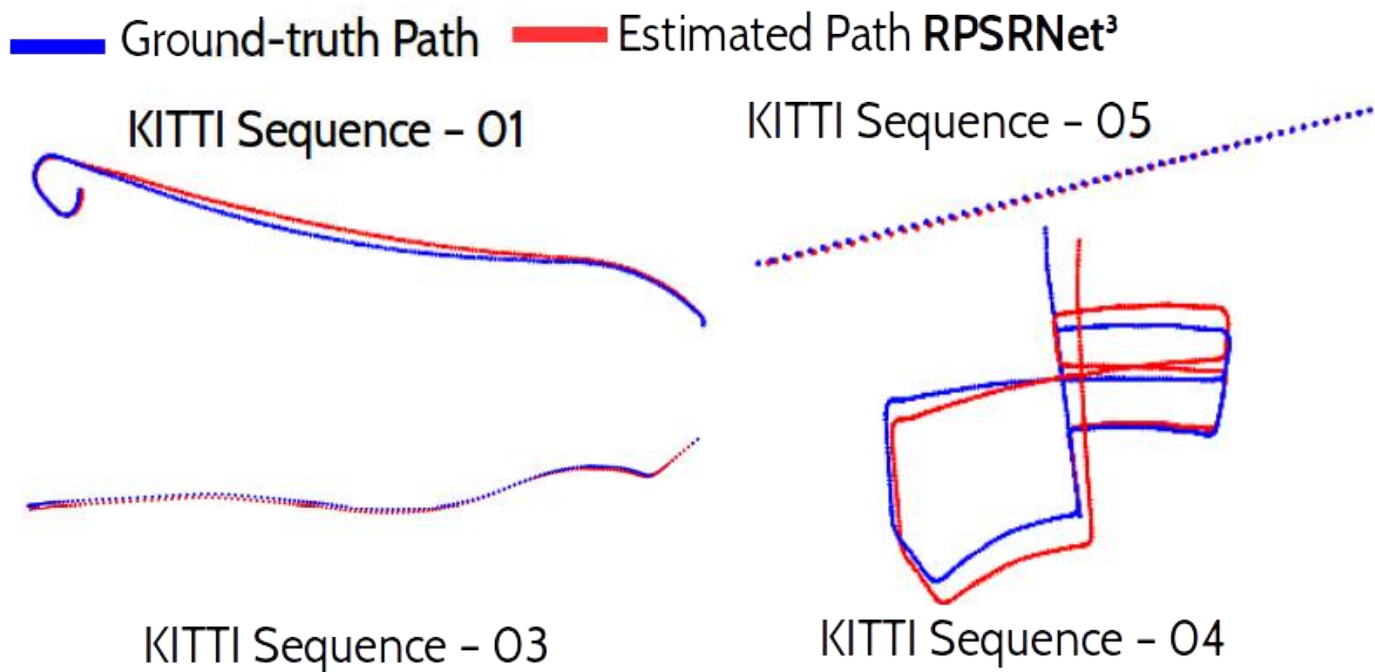
								Single Iteration	Three Iterations
	CPD [44]	GA* [25]	FGR [77]	ICP [10]	FilterReg [21]	DCP-v2 [66]	PointNetLK [5]	RPSRNet <sup>1</sup> (ours)	RPSRNet <sup>3</sup> (ours)
Seq.	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$
mean	3.55, 1.08	3.30, 1.0	3.29, 0.85	3.15, 1.08	3.08, 0.77	2.92, 0.89	4.02, 1.12	3.13, 0.88	<b>2.22, 0.58</b>
	3.03, 1.07	2.94, 1.02	3.25, 1.11	3.06, 1.20	3.26, 1.20	2.96, <b>0.76</b>	5.17, 1.20	3.03, 1.01	<b>2.18, 0.84</b>



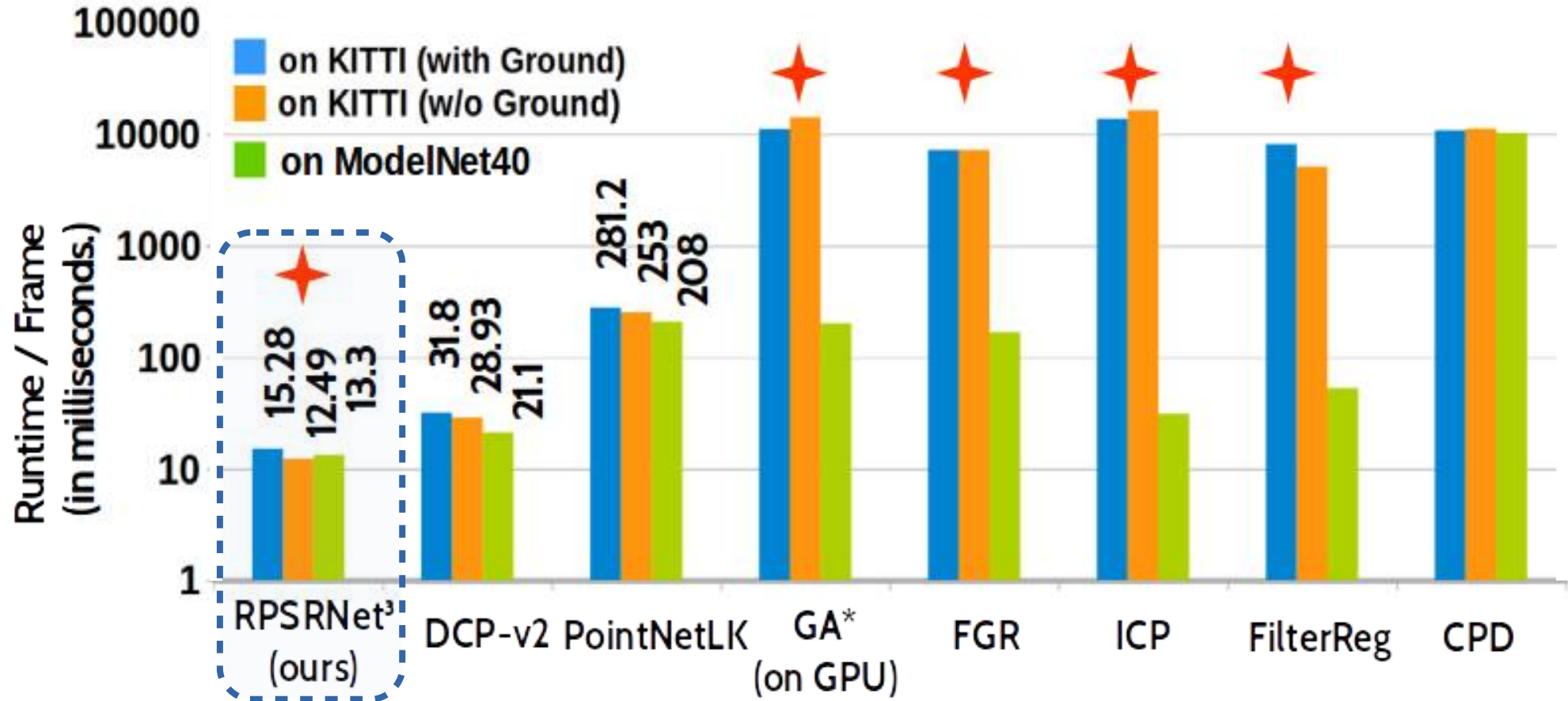
— Ground-truth Path — Estimated Path RPSRNet<sup>3</sup>

# KITTI LiDAR Odometry (Without Pose Graph Optimization and Loop Closure)

							Single Iteration	Three Iterations	
	CPD [44]	GA* [25]	FGR [77]	ICP [10]	FilterReg [21]	DCP-v2 [66]	PointNetLK [5]	RPSRNet <sup>1</sup> (ours)	RPSRNet <sup>3</sup> (ours)
Seq.	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$
mean	3.55, 1.08	3.30, 1.0	3.29, 0.85	3.15, 1.08	3.08, 0.77	2.92, 0.89	4.02, 1.12	3.13, 0.88	<b>2.22, 0.58</b>
	3.03, 1.07	2.94, 1.02	3.25, 1.11	3.06, 1.20	3.26, 1.20	2.96, <b>0.76</b>	5.17, 1.20	3.03, 1.01	<b>2.18, 0.84</b>



# Runtime Evaluation On KITTI and ModelNet40 Datasets (*Clean*)



★ Using Original Point Size (i.e., without sub-sampling)

\*\*BH-Tree Construction time ~4 ms

**Thanks for Watching!**